Hecke Operators on Modular Forms with Eta-Multiplier

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September 7, 2021

Abstract

The Dedekind-eta function is defined as $\eta(z) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$ where $q = e^{2\pi i/z}$. An eta-quotient of level N is $f_{N,\mathbf{v}}(z) = \prod_{\delta|N}\eta(\delta z)^{r_\delta}$, where $\mathbf{v} = \langle r_\delta : \delta \mid N\rangle \in \mathbb{Z}^{d(N)}$. For a suitable $D \mid 24$ and $\mathbf{v}$, we have that $f_{N,\mathbf{v}}(Dz)$ is a modular form of weight $k = \frac{1}{2}\sum_{\delta|N} r_\delta$ and character ξ. Let $\mathcal{A}_{N,\mathbf{v},w,\chi} = \{f_{N,\mathbf{v}}(Dz)F(Dz) : F(z) \in M_w(\Gamma_0(N), \chi)\} \subseteq M_{w+k}(\Gamma_0(D^2N), \xi\chi)$. In this talk, we show how the Hecke operator T_ℓ permutes these subspaces when $\ell \geq 5$ is prime, $N \in \{1, 2, 3, 4, 5, 6, 8, 9\}$, and $f_{N,\mathbf{v}}$ is a minimal, holomorphic, weight k, level N, eta-quotient of denominator D and character ξ. More precisely, we show that $T_\ell : \mathcal{A}_{N,\mathbf{v},w,\chi} \to \mathcal{A}_{N,\mathbf{v}',w+k-k',\chi\chi'}$ where $f_{N,\mathbf{v}'}(z)$ is a minimal, holomorphic, weight k', level N, eta-quotient of denominator D and character ξ'.